K_{l4} decays

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Abstract

An effective theory of large N_C QCD of mesons has been used to study six K_{l4} decay modes. It has been found that the matrix elements of the axial-vector current dominate the K_{l4} decays. PCAC is satisfied. A relationship between three form factors of axial-vector current has been found. Non-zero phase shifts are originated in $\rho \to \pi\pi$. The decay rates are calculated in the chiral limit. In this study there is no adjustable parameter.

1 Introduction

There is rich physics in kaon decays. Study on rare kaon decays are still active. The theoretical study of K_{l4} decays has a long history[1,2].

In Ref.[3] we have proposed an effective theory of large N_C QCD[4] of mesons. In this theory the diagrams at the tree level are at the leading order in large N_C expansion and the loop diagrams of mesons are at higher orders. This theory is phenomenologically successful[5,6,7]. We have used this theory to study K_{l3}[3], $K \to e\nu\gamma$[5,8], kaon form factors[7], and πK scattering[6] in the chiral limit. Theoretical results agree well with data. In these studies VMD and PCAC are satisfied. There are five parameters in this theory: three current quark masses, a parameter related to the quark condensate, and a universal coupling constant g which is determined to be 0.39 by fitting $\rho \to ee^+$. All parameters have been fixed by previous studies.

In this paper we use this theory of pseudoscalar, vector, and axial vector mesons[3] to study $K^- \to \pi^+\pi^- l\nu, \pi^0\pi^0 l\nu$, and $K_L \to \pi^\pm \pi^0 l^\mp \nu$. In the study of K_{l4} there is no adjustable parameter.

The Lagrangian of the theory of Ref. 3Y is

$$\mathcal{L} = \bar{\psi}(x)(i\gamma\cdot\partial + \gamma\cdot v + \gamma\cdot a\gamma_5 - mu(x))\psi(x) + \frac{1}{2}m_1^2(\rho_i^\mu\rho_{\mu i} + \omega^\mu\omega_\mu + a_i^\mu a_{\mu i} + f^\mu f_\mu)$$
$$+\frac{1}{2}m_2^2(K_\mu^{*a}\bar{K}^{*a\mu} + K_1^\mu K_{1\mu}) + \frac{1}{2}m_3^2(\phi_\mu\phi^\mu + f_s^\mu f_{s\mu})$$

$$+\bar{\psi}(x)_L\gamma\cdot W\psi(x)_L+\mathcal{L}_W+\mathcal{L}_{lepton}-\bar{\psi}M\psi, \quad (1)$$

where $a_\mu = \tau_i a^i_\mu + \lambda_a K^a_{1\mu} + (\frac{2}{3}+\frac{1}{\sqrt{3}}\lambda_8)f_\mu + (\frac{1}{3}-\frac{1}{\sqrt{3}}\lambda_8)f_{s\mu}(i = 1,2,3$ and $a = 4,5,6,7)$, $v_\mu = \tau_i\rho^i_\mu + \lambda_a K^*_\mu + (\frac{2}{3}+\frac{1}{\sqrt{3}}\lambda_8)\omega_\mu + (\frac{1}{3}-\frac{1}{\sqrt{3}}\lambda_8)\phi_\mu$, W^i_μ is the W boson, and $u = exp\{\gamma_5 i(\tau_i\pi_i + \lambda_a K^a + \eta + \eta')\}$, m is a parameter, and M is the mass matrix of u, d, s quarks, The masses m_1^2, m_2^2, and m_3^2 have been determined theoretically.

Using the notations of Ref.[1], we have

$$< \pi^i\pi^j|A_\mu|K >= \frac{i}{m_K}\{(p_1+p_2)_\mu F^{ij} + (p_1-p_2)_\mu G^{ij} + q_\mu R^{ij}\},$$
$$< \pi^i\pi^j|V_\mu|K >= \frac{H^{ij}}{m_K^3}\varepsilon^{\mu\nu\lambda\rho}p_\nu(p_1+p_2)_\lambda(p_1-p_2)_\rho, \quad (2)$$

where p_1, p_2, p are momenta of two pions and kaon respectively, $q = p - p_1 - p_2$, and $i, j = +, -, 0$. We define

$$q_1^2 = (p-p_1)^2, \quad q_2^2 = (p-p_2)^2, \quad q_3^2 = (p_1+p_2)^2.$$

The form factors, F^{ij}, G^{ij}, R^{ij} and H^{ij} are functions of q^2, q_1^2, q_2^2, and q_3^2. These four variables satisfy

$$q_1^2 + q_2^2 + q_3^2 = m_K^2 + 2m_\pi^2 + q^2.$$

The paper is organized as: 1)introduction; 2)isospin relation; 3)form factors of vector current; 4)$K^* \to K\pi\pi$ decay; 5)form factors of axial-vector current; 6)decay rates; 7)conclusions.

2 Isospin relation

For the decay modes $K^- \to \pi^+\pi^- l\nu, \pi^0\pi^0 l\nu$ and $\bar{K^0} \to \pi^+\pi^0 l\nu$ there are isospin relations between the form factors denoted as A^{ij}. We take -π^+, π^0, and π^- as isospin triplet and -$\bar{K}^0$ and K^- as isospin doublet. The isospin relation is obtained as

$$A^{+-} = A^{00} - \frac{1}{\sqrt{2}} A^{+0}, \tag{3}$$

where $A^{ij} = F^{ij}, G^{ij}, R^{ij}, H^{ij}$ respectively.

3 Form factors of vector current

The Vector Meson Dominance(VMD) is a natural result of this theory[3]. The coupling between the W bosons and the bosonized vector current($\Delta s = 1$) has been derived as[5]

$$\begin{aligned} \mathcal{L}^V &= \frac{g_W}{4} sin\theta_C g\{-\frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial_\mu K_\nu^{*-} - \partial_\nu K_\mu^{*-}) - \frac{1}{2}(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-)(\partial_\mu K_\nu^{*+} & (4) \\ &\quad -\partial_\nu K_\mu^{*+}) + W_\mu^+ j_\mu^- + W_\mu^- j_\mu^+\}, & (5) \end{aligned}$$

where $j_\mu^\pm$ is obtained by substituting

$$K_\mu^\pm \to \frac{g_W}{4} sin\theta_C g W_\mu^\pm$$

into the vertex in which K_μ field is involved.

The matrix elements of the vector current of K_{l4} are resulted in anomalous vertices of mesons. The two subprocesses are shown in Fig.1(a,b). There is contact term. Three kinds

of vertices are involved: the contact term $\mathcal{L}_{K^*K\pi\pi}$, $\mathcal{L}_{K^*K^*\pi}$ and $\mathcal{L}_{K^*K\pi}$, and $\mathcal{L}_{K^*K\rho}$ and $\mathcal{L}_{\rho\pi\pi}$. In the chiral limit, $m_q \to 0$, all these vertices have been derived from the Lagrangian (1) [3] and are listed below

$$\begin{aligned}
\mathcal{L}_{K^*K^*\pi} &= -\frac{N_C}{\pi^2 g^2 f_\pi}\ \varepsilon^{\mu\nu\alpha\beta} d_{aci} K^a_\mu \partial_\nu K^c_\alpha \partial_\beta \pi^i \\
\mathcal{L}_{K^*K\pi} &= \frac{2}{g} f(q^2) f_{abi} K^a_\mu (\partial_\mu \pi^i K^b - \pi^i \partial_\mu K^b), \\
f(q^2) &= 1 + \frac{q^2}{2\pi^2 f_\pi^2}[(1-\frac{2c}{g})^2 - 4\pi^2 c^2)], \\
c &= \frac{f_\pi^2}{2g m_\rho^2}, \\
\mathcal{L}_{K^*\rho K} &= -\frac{N_C}{\pi^2 g^2 f_\pi^2} \varepsilon^{\mu\nu\alpha\beta} d_{abi} K^a_\mu \partial_\nu \rho^i_\alpha \partial_\beta K^b, \\
\mathcal{L}_{\rho\pi\pi} &= \frac{2}{g} f(q^2) \epsilon_{ijk} \rho^i_\mu \pi^j \partial_\mu \pi^k, \qquad (6) \\
\mathcal{L}_{K^*K\pi\pi} &= \frac{2}{g\pi^2 f_\pi^3}(1 - \frac{6c}{g} + \frac{6c^2}{g^2}) d_{abe} f_{cde} \varepsilon^{\mu\nu\alpha\beta} K^a_\mu \partial_\nu P^b \partial_\alpha P^c \partial_\beta P^d, \qquad (7)
\end{aligned}$$

From theses vertices the form factors h^{ij} are found

$$\begin{aligned}
H^{+-} &= \frac{m_K^3 m_{K^*}^2}{q^2 - m_{K^*}^2}\{\frac{1}{\pi^2 f_\pi^3}(1 - \frac{6c}{g} + \frac{6c^2}{g^2}) - \frac{N_C}{g^2\pi^2 f_\pi}\frac{f(q_2^2)}{q_2^2 - m_{K^*}^2} \\
&- \frac{N_C}{2g^2\pi^2 f_\pi}\frac{f(q_3^2)}{q_3^2 - m_\rho^2 + i\sqrt{q_3^2}\Gamma(q_3^2)}\}, \qquad (8) \\
H^{00} &= -\frac{m_K^3 m_{K^*}^2}{q^2 - m_{K^*}^2}\frac{N_c}{2g^2\pi^2 f_\pi}\{\frac{f(q_2^2)}{q_2^2 - m_{K^*}^2} - \frac{f^2(q_1^2)}{q_1^2 - m_{K^*}^2}\}, \qquad (9) \\
H^{+0} &= \frac{1}{\sqrt{2}}\frac{m_K^3 m_{K^*}^2}{q^2 - m_{K^*}^2}\{-\frac{2}{\pi^2 f_\pi^3}(1 - \frac{6c}{g} + \frac{6c^2}{g^2}) + \frac{N_C}{\pi^2 g^2 f_\pi}[\frac{f(q_1)}{q_1^2 - m_{K^*}^2} + \frac{f(q_2)}{q_2^2 - m_{K^*}^2} \\
&+ \frac{f(q_3)}{q_3^2 - m_\rho^2 + i\sqrt{q_3^2}\Gamma_\rho(q_3^2)}]\}, \qquad (10)
\end{aligned}$$

where Γ_ρ is the decay width of ρ meson

$$\Gamma_\rho(q_3^2) = \frac{\sqrt{q_3^2} f^2(q_3^2)}{12g^2\pi}(1 - \frac{4m_\pi^2}{q_3^2})^{\frac{3}{2}}. \tag{11}$$

The equations(8-10) show that the isospin relation(3) is satisfied.

4 $K^* \to K\pi\pi$ decay

The vertices(6,7) are responsible for the decay of $K^* \to K\pi\pi$. As a test the decay widths of $K^* \to K\pi\pi$ are calculated

$$\Gamma(K^{*-} \to K^-\pi^+\pi^-) = \frac{1}{96(2\pi)^3 m_{K^*}} \int dk_1^2 dk_2^2 \{p_1^2 p_2^2 - (\vec{p}_1 \cdot \vec{p}_2)^2\}|A|^2 = 0.29 \times 10^{-5} GeV \tag{12}$$

which is less than the experimental upper limit[9], where A is the amplitude

$$\begin{aligned} A &= \frac{4}{g\pi^2 f_\pi^3}(1 - \frac{6c}{g} + \frac{6c^2}{g^2}) - \frac{4N_c}{g^3\pi^2 f_\pi}\frac{f(k_2^2)}{k_2^2 - m_{K^*}^2 + i\sqrt{k_2^2}\Gamma_{K^*}(k_2^2)} \\ &- \frac{2N_c}{g^3\pi^2 f_\pi}\frac{f(k_3^2)}{k_3^2 - m_\rho^2 + i\sqrt{k_3^2}\Gamma_\rho(k_3^2)} \end{aligned} \tag{13}$$

where $k_1^2 = (p+p_1)^2$, $k_2^2 = (p+p_2)^2$, $k_3^2 = (p_1+p_2)^2$, and p_1, p_2, p are momenta of π^+, π^- and K^- respectively, Γ_{K^*} is the decay width of K^*

$$\Gamma_{K^*}(k_2^2) = \frac{f^2(k_2^2)}{2\pi g^2 k_2^2}\{\frac{1}{4k_2^2}(k_2^2 + m_K^2 - m_\pi^2)^2 - m_K^2\}^{\frac{3}{2}}. \tag{14}$$

$$\Gamma(K^{*-} \to K^-\pi^0\pi^0) = \frac{1}{192(2\pi)^3 m_{K^*}} \int dk_1^2 dk_2^2 \{p_1^2 p_2^2 - (\vec{p}_1 \cdot \vec{p}_2)^2\}$$

$$
\frac{36}{\pi^4 g^6 f_\pi^2}\{\frac{f(k_1)}{k_1^2-m_{K^*}^2+i\sqrt{k_1^2}\Gamma_{K^*}(k_1^2)}-\frac{f(k_2)}{k_2^2-m_{K^*}^2+i\sqrt{k_2^2}\Gamma_{K^*}(k_2^2)}\}^2
$$
$$
=0.61\times 10^{-6}GeV. \tag{15}
$$

$$
\Gamma(K^{*-}\to\bar{K}^0\pi^-\pi^0)=\frac{1}{96(2\pi)^3 m_{K^*}}\int dk_1^2 dk_2^2\{p_1^2p_2^2-(\vec{p}_1\cdot\vec{p}_2)^2\}|B|^2=0.38\times 10^{-4}GeV, \tag{16}
$$

where

$$
B=-\frac{8}{\sqrt{2}gf_\pi^3}(1-\frac{6c}{g}+\frac{6c^2}{g^2})+\frac{12}{\sqrt{2}\pi^2g^3f_\pi}\{\frac{f(k_1)}{k_1^2-m_{K^*}^2+i\sqrt{k_1^2}\Gamma_{K^*}(k_1^2)}
$$
$$
+\frac{f(k_2)}{k_2^2-m_{K^*}^2+i\sqrt{k_2^2}\Gamma_{K^*}(k_2^2)}+\frac{f(k_3)}{k_3^2-m_\rho^2+i\sqrt{k_3^2}\Gamma_\rho(k_3^2)}\}. \tag{17}
$$

Eq.(16) is compatible with the data 9Y.

5 Form factors of axial-vector current

In the chiral limit, the axial-vector part of the interaction between W-boson and mesons is expressed as[5]

$$
\mathcal{L}^{As}=\frac{g_W}{4}\frac{1}{f_a}sin\theta_C\{-\frac{1}{2}(\partial_\mu W_\nu^\pm-\partial_\nu W_\mu^\pm)(\partial^\mu K_a^{\mp\nu}-\partial^\nu K_a^{\mp\mu})+W^{\pm\mu}j_\mu^\mp\}
$$
$$
+\frac{g_W}{4}sin\theta_C\Delta m^2 f_a W_\mu^\pm K^{\mp\mu}+\frac{g_w}{4}\sin\theta_C f_K W_\mu^\pm\partial^\mu K^\mp, \tag{18}
$$

where $j_\mu^\pm$ are obtained by substituting $K_{a\mu}^\pm\to\frac{g_W}{4f_a}sin\theta_C W_\mu^\pm$ into the vertex in which K_a fields are involved,

$$
f_a=g^{-1}(1-\frac{1}{2\pi^2g^2})^{-\frac{1}{2}}, \tag{19}
$$

$$\Delta m^2 = 6m^2 g^2 = f_\pi^2 (1 - \frac{f_\pi^2}{g^2 m_\rho^2})^{-1}, \tag{20}$$

$$c = \frac{f_\pi^2}{2g m_\rho^2}. \tag{21}$$

The mass of K_1 meson is determined by

$$(1 - \frac{1}{2\pi^2 g^2}) m_{K_1}^2 = 6m^2 + m_{K^*}^2. \tag{22}$$

The numerical value is $m_{k_1} = 1.322GeV$ which is compatible with the data [9].

Two subprocesses contribute to the matrix element of the axial-vector current. They are shown in Fig.2(a,b). The vertices of mesons involved in these processes are $\mathcal{L}_{K_1 K^* \pi}$, $\mathcal{L}_{K^* K \pi}$ and $\mathcal{L}_{K_1 \rho K}$, $\mathcal{L}_{\rho\pi\pi}$. There is a contact term $\mathcal{L}_{K_1 K\pi\pi}$ too. However, the calculation shows that the contribution of the contact term is very small and negligible. In the chiral limit, these vertices have been derived from the Lagrangian(1)

$$\mathcal{L}_{K_1 K^* \pi} = f_{abi}\{A(p^2) K_{1\mu}^a K_\mu^{*b} \pi^i - B K_{1\mu}^a K_\nu^{*b} \partial_{\mu\nu} \pi^i + D K_{1\mu}^a \partial^\mu (K_\nu^{*b} \partial^\nu \pi^i)\} \tag{23}$$

$$\mathcal{L}_{K_1 \rho K} = -f_{abi}\{A(p^2) K_{1\mu}^a \rho_\mu^i K^b - B K_{1\mu}^a \rho_\nu^i \partial_{\mu\nu} K^b + D K_{1\mu}^a \partial^\mu (\rho_\nu^i \partial^\nu K^b)\}, \tag{24}$$

where

$$A(p^2) = \frac{2}{f_\pi} g f_a \{\frac{F^2}{g^2} + p^2 [\frac{2c}{g} + \frac{3}{4\pi^2 g^2}(1 - \frac{2c}{g})]$$
$$+ q^2 [\frac{1}{2\pi^2 g^2} - \frac{2c}{g} - \frac{3}{4\pi^2 g^2}(1 - \frac{2c}{g})]\}, \tag{25}$$

$$F^2 = f_\pi^2 (1 - \frac{2c}{g})^{-1}, \tag{26}$$

$$B = -\frac{2}{f_\pi} g f_a \frac{1}{2\pi^2 g^2}(1 - \frac{2c}{g}), \tag{27}$$

$$D = -\frac{2}{f_\pi} f_a \{2c + \frac{3}{2\pi^2 g}(1 - \frac{2c}{g})\}, \tag{28}$$

where q and p are the momentum of K_1 and the vector meson respectively.

$$\mathcal{L}_{K^* K\pi} = \frac{2}{g} f_{abi} f(p^2) K^a_\mu (K^b \partial_\mu \pi^i - \pi^i \partial_\mu K^b), \tag{29}$$

$$\mathcal{L}_{\rho\pi\pi} = \frac{2}{g} \epsilon_{ijk} f(p^2) \rho^i_\mu \pi^j \partial_\mu \pi^k, \tag{30}$$

$$f(p^2) = 1 + \frac{p^2}{2\pi f_\pi^2}[(1 - \frac{2c}{g})^2 - 4\pi^2 c^2], \tag{31}$$

where p is the momentum of the vector meson.

By using Eqs.(18,23,24), we obtain

$$\begin{aligned} < \pi^+\pi^-|A_\mu|K^- > \quad = \quad & \frac{1}{\sqrt{2}}(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu})\frac{g^2 f_a m^2_{K^*}}{q^2 - m^2_{K_1}} < \pi^+\pi^-|\{A(p_{K^*})\bar{K^0}_\nu \pi^- - B\bar{K^0}_\lambda \partial_{\lambda\nu}\pi^-\} \\ & -\frac{1}{\sqrt{2}}\{A(p_\rho)\rho^0_\nu K^- - B\rho^0_\lambda \partial_{\lambda\nu} K^-\}|K^- > . \end{aligned} \tag{32}$$

In the chiral limit PCAC is satisfied. The reason is that the Lagrangian(1) is chiral symmetric in the limit $m_q \rightarrow 0$. On the other hand, the satisfaction of PCAC is resulted in the cancellations between the four terms of Eq.(18). The Eq.(18) shows that the axial-vector current has more complicated structure than the vector current does(5). Because of the PCAC the form factor R(2) is not an independent quantity and determined as

$$R = -\frac{1}{q^2}\{q \cdot (p_1 + p_2)F + q \cdot (p_1 - p_2)G\}. \tag{33}$$

Substituting the vertices(29,30) into Eq.(32), the three form factors are obtained

$$F^{+-} = \frac{gf_a m_{K^*}^2 m_K}{q^2 - m_{K_1}^2}\{\frac{f(q_2^2)}{q_2^2 - m_{K^*}^2}[\frac{3}{2}A(q_2^2) + \frac{1}{2}Bp_1 \cdot (p + p_2)]$$
$$+\frac{f(q_3^2)}{q_3^2 - m_\rho^2 + i\sqrt{q_3^2}\Gamma_\rho(q_3^2)}Bp \cdot (p_2 - p_1)\}, \tag{34}$$
$$G^{+-} = \frac{gf_a m_{K^*}^2 m_K}{q^2 - m_{K_1}^2}\{\frac{f(q_2^2)}{q_2^2 - m_{K^*}^2}[-\frac{1}{2}A(q_2^2) + \frac{1}{2}Bp_1 \cdot (p + p_2)]$$
$$-\frac{f(q_3^2)}{q_3^2 - m_\rho^2 + i\sqrt{q_3^2}\Gamma_\rho(q_3^2)}A(q_3^2)\}. \tag{35}$$

In the same way the form factors of other two decay modes are obtained

$$F^{00} = \frac{1}{2}\frac{gf_a m_{K^*}^2 m_K}{q^2 - m_{K_1}^2}\{\frac{f(q_1^2)}{q_1^2 - m_{K^*}^2}[\frac{3}{2}A(q_1^2) + \frac{1}{2}B(p_2 \cdot p + p_2 \cdot p_1)]$$
$$+\frac{f(q_2^2)}{q_2^2 - m_{K^*}^2}[\frac{3}{2}A(q_2^2) + \frac{1}{2}B(p_1 \cdot p + p_1 \cdot p_2)]\}, \tag{36}$$
$$G^{00} = \frac{1}{2}\frac{gf_a m_{K^*}^2 m_K}{q^2 - m_{K_1}^2}\{\frac{f(q_1^2)}{q_1^2 - m_{K^*}^2}[\frac{1}{2}A(q_1^2) - \frac{1}{2}B(p_2 \cdot p + p_2 \cdot p_1)]$$
$$+\frac{f(q_2^2)}{q_2^2 - m_{K^*}^2}[-\frac{1}{2}A(q_2^2) + \frac{1}{2}B(p_1 \cdot p + p_1 \cdot p_2)]\}, \tag{37}$$
$$F^{+0} = \frac{1}{\sqrt{2}}\frac{gf_a m_{K^*}^2 m_K}{q^2 - m_{K_1}^2}\{\frac{f(q_1^2)}{q_1^2 - m_{K^*}^2}[\frac{3}{2}A(q_1^2) + \frac{1}{2}B(p_2 \cdot p + p_2 \cdot p_1)]$$
$$-\frac{f(q_2^2)}{q_2^2 - m_{K^*}^2}[\frac{3}{2}A(q_2^2) + \frac{1}{2}B(p_1 \cdot p + p_1 \cdot p_2)]$$
$$+\frac{2f(q_3^2)}{q_3^2 - m_\rho^2 + i\sqrt{q_3^2}\Gamma_\rho(q_3^2)}Bp \cdot (p_1 - p_2)\}, \tag{38}$$
$$G^{+0} = \frac{1}{\sqrt{2}}\frac{gf_a M_{K^*}^2 m_K}{q^2 - m_{K_1}^2}\{\frac{f(q_1^2)}{q_1^2 - m_{K^*}^2}[\frac{1}{2}A(q_1^2) - \frac{1}{2}B(p_2 \cdot p + p_2 \cdot p_1)]$$
$$-\frac{f(q_2^2)}{q_2^2 - m_{K^*}^2}[-\frac{1}{2}A(q_2^2) + \frac{1}{2}B(p_1 \cdot p + p_1 \cdot p_2)] + \frac{2f(q_3^2)}{q_3^2 - m_\rho^2 + i\sqrt{q_3^2}\Gamma_\rho(q_3^2)}A(q_3^2)\}. \tag{39}$$

The isospin relations(3) between these form factors are satisfied.

The partial wave analysis of these form factors can be done. The decay channel $\rho \to \pi\pi$ contributes to the decay modes of $\pi^+\pi^-$ and $\pi^+\pi^0$. The range of the variable q_3^2 is $4m_\pi^2 < q_3^2 < (m_K - m_l)^2$ in which the decay width $\Gamma_\rho(q_3^2)$ is not zero. The form factors, A^{+-} and A^{+0} are complex functions of q_3^2. The $\rho \to \pi\pi$ doesn't contribute to $\pi^0\pi^0$ mode. Therefore, F^{00} and G^{00} are real. K_{l4} are decays at low energies. s- and p- waves are major partial waves. The q_1^2 and q_2^2 variables are expressed as

$$q_1^2 = \frac{1}{2}(m_K^2 + 2m_\pi^2 + q^2 - q_3^2) + (1 - \frac{4m_\pi^2}{q_3^2})^{\frac{1}{2}} X cos\theta_\pi, \tag{40}$$

$$q_2^2 = \frac{1}{2}(m_K^2 + 2m_\pi^2 + q^2 - q_3^2) - (1 - \frac{4m_\pi^2}{q_3^2})^{\frac{1}{2}} X cos\theta_\pi, \tag{41}$$

where $X = \{\frac{1}{4}(m_K^2 - q^2 - q_3^2)^2 - q^2 q_3^2\}^{\frac{1}{2}}$ and θ_π is the angle between $\vec{p}_1$ and $\vec{p}$ in the rest frame of the two pions.

The s- and p- wave amplitudes are obtained from Eqs.(34-39)

1. F_s^{+-} is real. Only Fig.2(a) contributes to it. F_p^{+-} is a complex function of q_3^2 resulted by $\rho \to \pi\pi$. F_p^{+-} has a phase shift.

$$F^{+-} = F_s^{+-} + |F_p^{+-}| e^{i\delta_p^{+-}} (1 - \frac{4m_\pi^2}{q_3^2})^{\frac{1}{2}} \frac{X}{m_K^2} cos\theta_\pi. \tag{42}$$

2. G_s^{+-} is complex and has a phase shift. G_p^{+-} is real.

$$G^{+-} = |G_s^{+-}| e^{i\delta_s^{+-}} + G_p^{+-} (1 - \frac{4m_\pi^2}{q_3^2})^{\frac{1}{2}} \frac{X}{m_K^2} cos\theta_\pi. \tag{43}$$

3. Both G_s^{00} and G_p^{00} are real.

$$
\begin{aligned}
F^{00} &= F_s^{00}, \\
G^{00} &= |G_p^{00}|(1-\frac{4m_\pi^2}{q_3^2})^{\frac{1}{2}}\frac{X}{m_K^2}cos\theta_\pi. \qquad (44)
\end{aligned}
$$

4. The isospin of the two pions of the $\pi^+\pi^0$ mode is one. Because of Bose statistics F^{+0} only has p-wave which is complex and has phase shift. G^{+0} has s wave only. G_s^{+0} is complex and it has phase shift.

$$
\begin{aligned}
F^{+0} &= |F_p^{+0}|e^{i\delta_p^{+0}}(1-\frac{4m_\pi^2}{q_3^2})^{\frac{1}{2}}\frac{X}{m_K^2}cos\theta_\pi, \\
G^{+0} &= |G_s^{+0}|e^{\delta_s^{+0}}. \qquad (45)
\end{aligned}
$$

All the phase shifts are caused by the decay $\rho \to \pi\pi$ and functions of q^2 and q_3^2.

6 Decay rates

The decay rates of the three modes of K_{e4} and $K_{\mu4}$ are calculated. As mentioned above, all the form factors are derived in the chiral limit. Therefore, only the leading terms of the masses of kaon and pions are kept in the calculation of the decay rates.

Ignoring m_e, only the form factors F, G, and H contribute to the decay rates of K_{e4}. By using the formula of Ref.[1] we obtain

$$\Gamma(K^- \to \pi^+\pi^- e\nu) = 2.06 \times 10^{-21} GeV, \quad B = 3.87 \times 10^{-5}.$$

$$\Gamma(K^- \to \pi^0\pi^0 e\nu) = 0.221 \times 10^{-21} GeV, \quad B = 0.42 \times 10^{-5}.$$

$$\Gamma(K^- \to \pi^+\pi^0 e\nu) = 3.24 \times 10^{-21} GeV, \quad B = 2.55 \times 10^{-4}.$$

The experimental data are

$$B(\pi^+\pi^-) = (3.91 \pm 0.17) \times 10^{-5}[10],$$

$$B(\pi^0\pi^0) = (2.54 \pm 0.89) \times 10^{-5}(10\ events)[11],$$

$$B(\pi^-\pi^0) = (5.16 \pm 0.20 \pm 0.22) \times 10^{-5}[12],$$

$$B(\pi^-\pi^0) = (6.2 \pm 2.0) \times 10^{-5}[13],$$

$$B(\pi^-\pi^0) < 200 \times 10^{-5}[14].$$

Theoretical result of $\pi^+\pi^-$ mode agrees well with the data.

The form factors of the vector current are determined by anomalous vertices. The numerical calculation shows that the contribution of the form factor H is only 0.5% of the total decay rate of $K^- \to \pi^+\pi^- e\nu$. Therefore, the axial-vector current dominates the K_{l4} decays.

As shown in Fig.2(a,b) there are two channels in K_{l4} decays. Numerical calculation of $K^- \to \pi^+\pi^- e\nu$ shows that the contribution of $\rho \to \pi\pi$(Fig.2(b)) is twice of the process, $K^* \to K\pi$, (Fig.2(a)). Only the process(Fig.2(a)) contribute to $K^- \to \pi^0\pi^0 e\nu$. Because of Bose statistics there is an additional factor of $\frac{1}{2}$ in the formula of the decay rate of this mode. Therefore, this theory predicts smaller decay rate for this decay mode. On the other

hand, the numerical calculation shows that the process(Fig.2(b)) is the major contributor of the decay $\bar{K}^0 \to \pi^+\pi^0 e\nu$. The theory predicts a larger branching ratio for $\bar{K}^0 \to \pi^+\pi^0 e\nu$. All the form factors contribute to $K_{\mu 4}$ decays. Eq.(33) shows that in the chiral limit PCAC predicts that the form factor R is determined by other two form factors, F and G. The branching ratio of $K_{\mu 4}$ provides a test on this prediction. The numerical results are

$$\Gamma(K^- \to \pi^+\pi^-\mu\nu) = 0.634 \times 10^{-21} GeV, \quad B = 1.19 \times 10^{-5}.$$

$$\Gamma(K^- \to \pi^0\pi^0\mu\nu) = 0.673 \times 10^{-22} GeV, \quad B = 0.126 \times 10^{-5}.$$

$$\Gamma(\bar{K}^0 \to \pi^+\pi^0\mu\nu) = 1.01 \times 10^{-21} GeV, \quad B = 0.793 \times 10^{-4}.$$

The experimental data[9] is

$$B(K^- \to \pi^+\pi^-\mu\nu) = (1.4 \pm 0.9) \times 10^{-5}.$$

Theory agrees with the data well.

7 Conclusion

All the four form factors of K_{l4} have been derived from an effective theory of large N_C QCD in the chiral limit. It has been found that the contribution of the vector current is negligible and the axial-vector current is dominant in K_{l4} decays. PCAC is revealed from the theory. In the chiral limit it has been predicted that the form factor R is determined by the form

factors F and G. The prediction has been tested by $K^- \to \pi^+\pi^-\mu\nu$. Theory agrees with the data. The partial wave analysis has been done. Non-zero phase shifts originate in the decay $\rho \to \pi\pi$. The process $K_1 \to \rho K$ and $\rho \to \pi\pi$(Fig.2(b)) plays important role in K_{l4} decays. Because of this channel the theory predicts larger branching ratio for $K^- \to \pi^+\pi^- e\nu$ and $\bar{K}^0 \to \pi^+\pi^0 e\nu$. The former agrees well with the data. ρ resonance doesn't contribute to $K^- \to \pi^0\pi^0 e\nu$. Therefore, the branching ratio of this decay mode is predicted to be smaller.

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Figure Captions

Fig.1 Feynman Diagrams of vector current

Fig.2 Feynman diagrams of axial-vector current

Fig.3 Phase shifts

Fig.4 Phase shifts **Fig.5** Phase shifts

Fig.5 Phase shifts

Fig.6 Phase shifts

Fig.7 Form factors od $\pi^+\pi^-$ mode

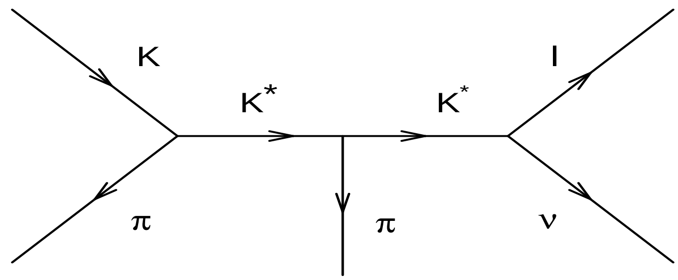
Fig.8 Form factors od $\pi^+\pi^-$ mode

Fig.9 Form factors od $\pi^+\pi^0$ mode

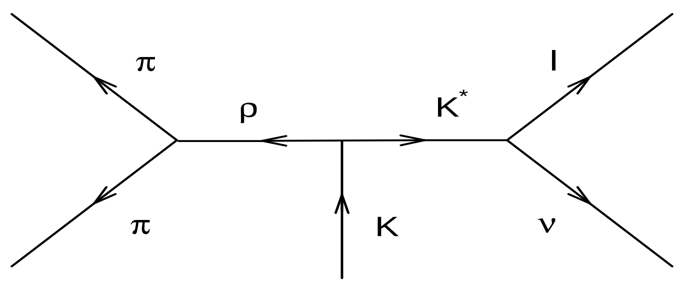
Fig.10 Form factors od $\pi^+\pi^0$ mode

Fig.11 Form factors od $\pi^0\pi^0$ mode

Fig.12 Form factors od $\pi^0\pi^0$ mode



(a)



(b)

Fig.1

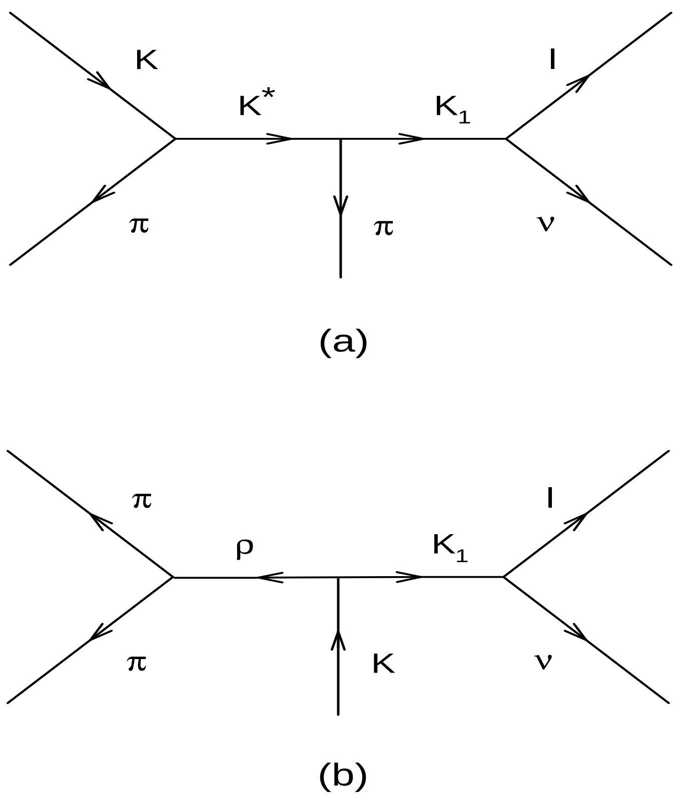
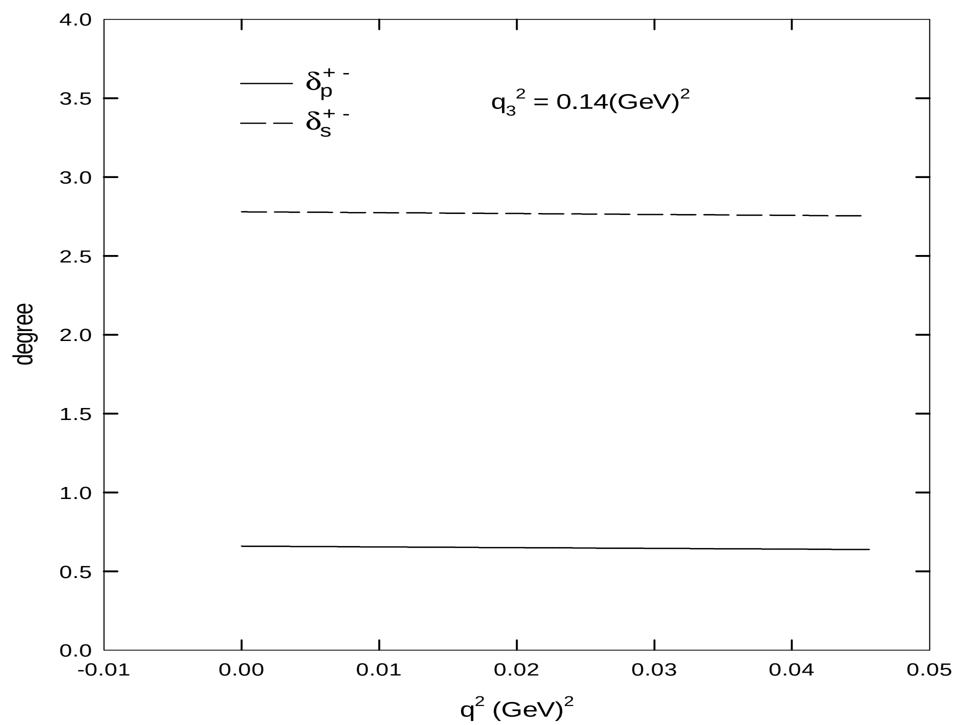


Fig.2



4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
degree
δ_p^{+-}
δ_s^{+-}
$q_3^2 = 0.14(GeV)^2$
-0.01
0.00
0.01
0.02
0.03
0.04
0.05
$q^2\ (GeV)^2$

Fig.3

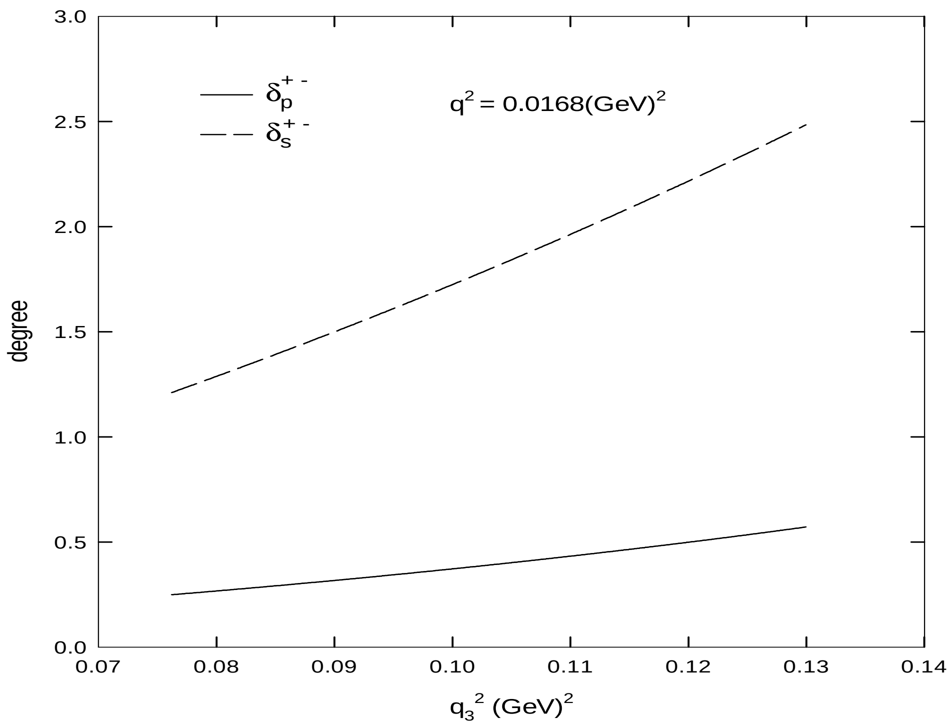
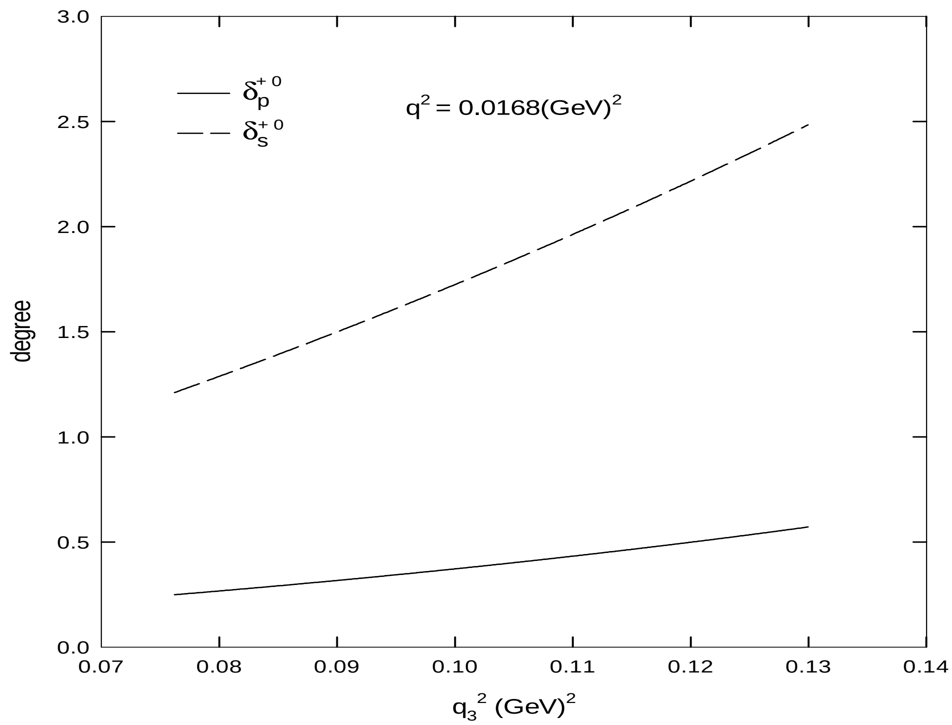
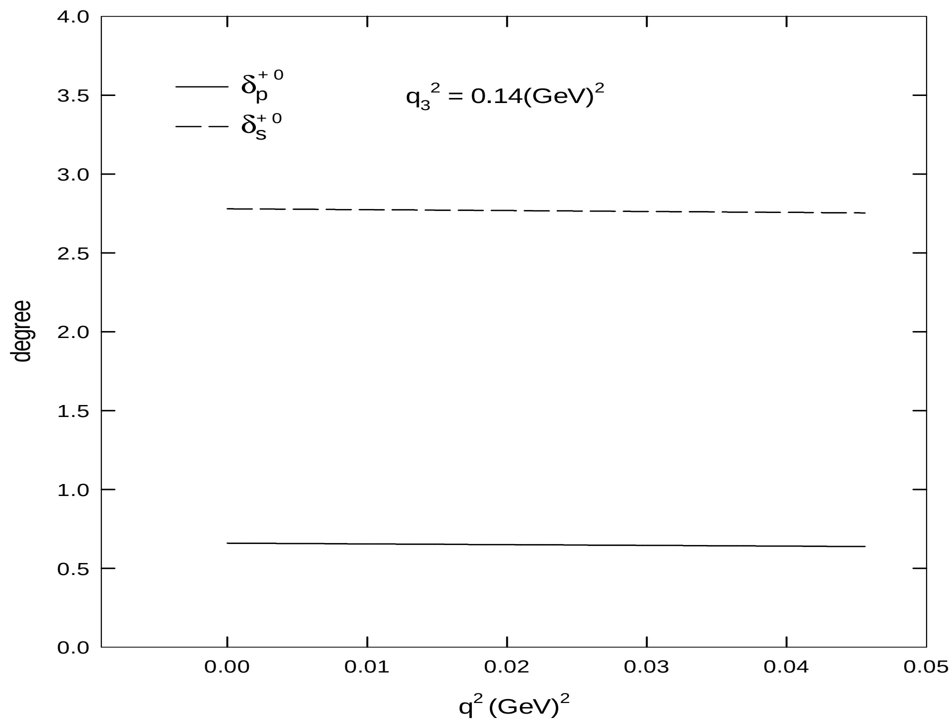

3.0
2.5
2.0
1.5
1.0
0.5
0.0
degree
δ_p^{+-}
δ_s^{+-}
$q^2 = 0.0168(GeV)^2$
0.07
0.08
0.09
0.10
0.11
0.12
0.13
0.14
q_3^2 (GeV)^2

Fig.4



3.0
2.5
2.0
1.5
1.0
0.5
0.0
degree
δ_p^{+0}
δ_s^{+0}
$q^2 = 0.0168(\mathrm{GeV})^2$
0.07
0.08
0.09
0.10
0.11
0.12
0.13
0.14
$q_3^2\ (\mathrm{GeV})^2$

Fig.5



4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
degree
δ_p^{+0}
δ_s^{+0}
$q_3^2 = 0.14(\mathrm{GeV})^2$
0.00
0.01
0.02
0.03
0.04
0.05
$q^2\,(\mathrm{GeV})^2$

Fig.6

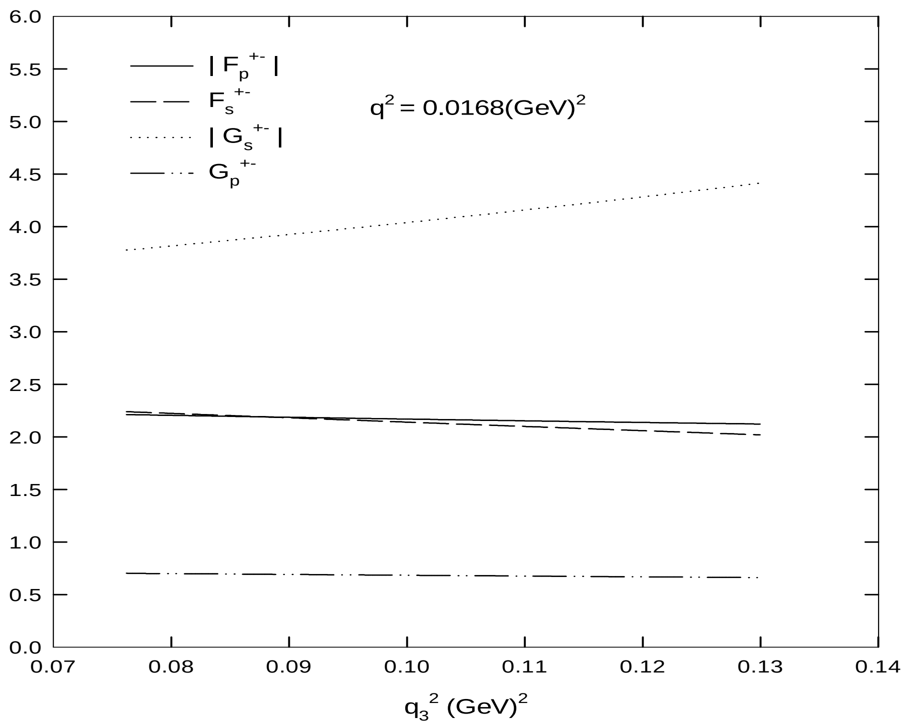

| F_p^{+-} |
F_s^{+-}
| G_s^{+-} |
G_p^{+-}
q^2 = 0.0168(GeV)^2
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.07
0.08
0.09
0.10
0.11
0.12
0.13
0.14
q_3^2 (GeV)^2

Fig.7

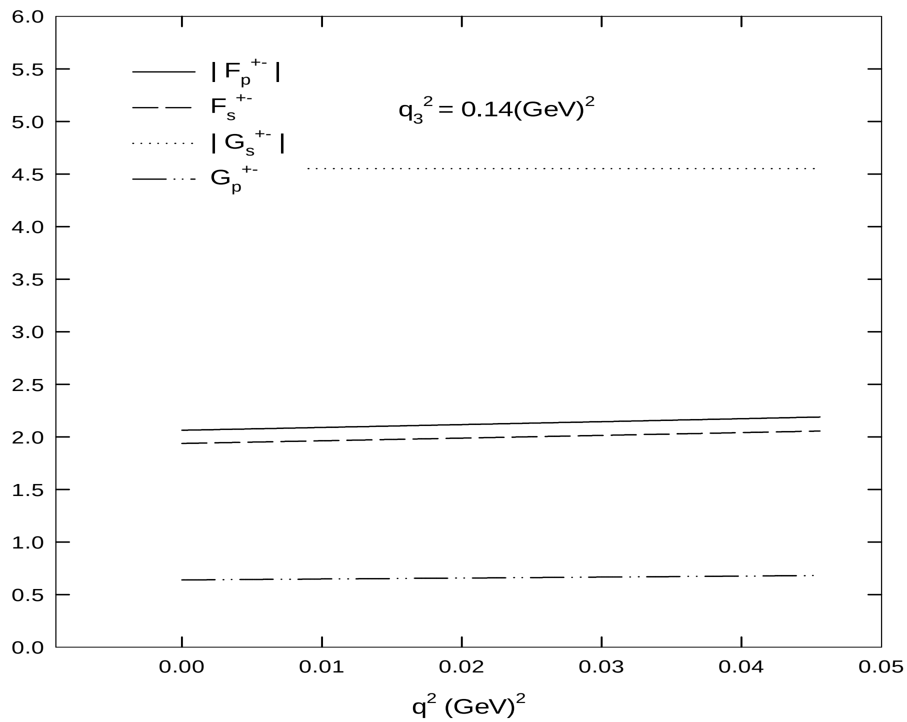
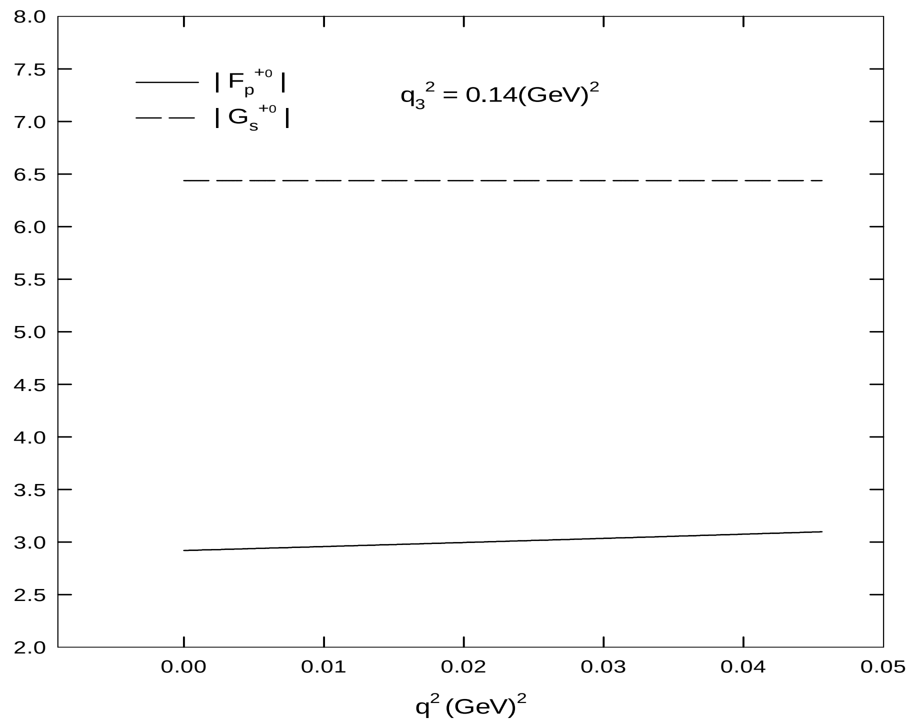
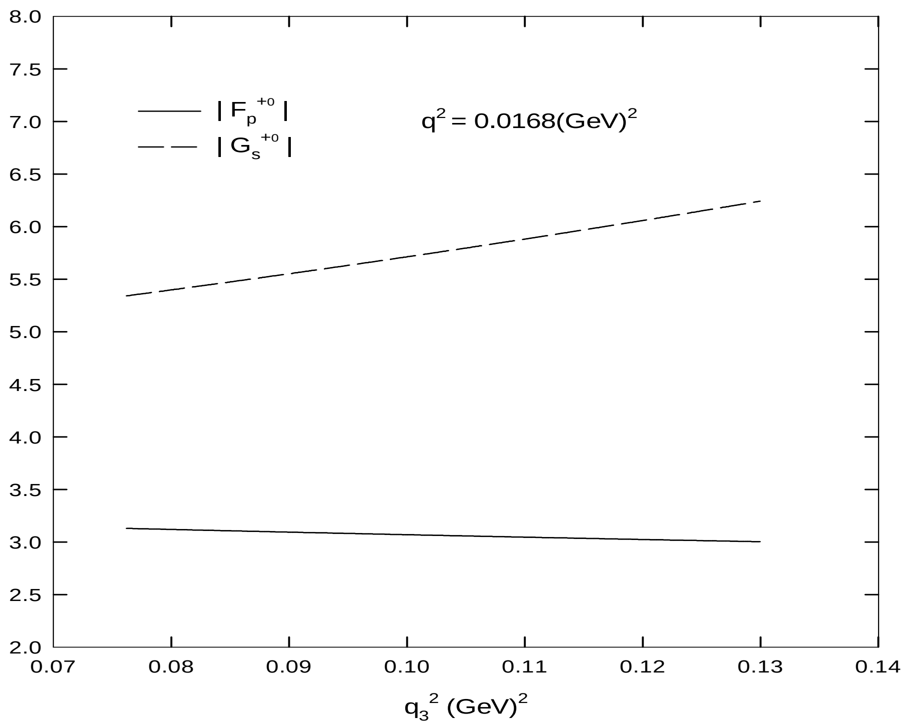

6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
| F_p^{+-} |
F_s^{+-}
| G_s^{+-} |
G_p^{+-}
$q_3^2 = 0.14(\mathrm{GeV})^2$
0.00
0.01
0.02
0.03
0.04
0.05
q^2 (GeV)^2

Fig.8



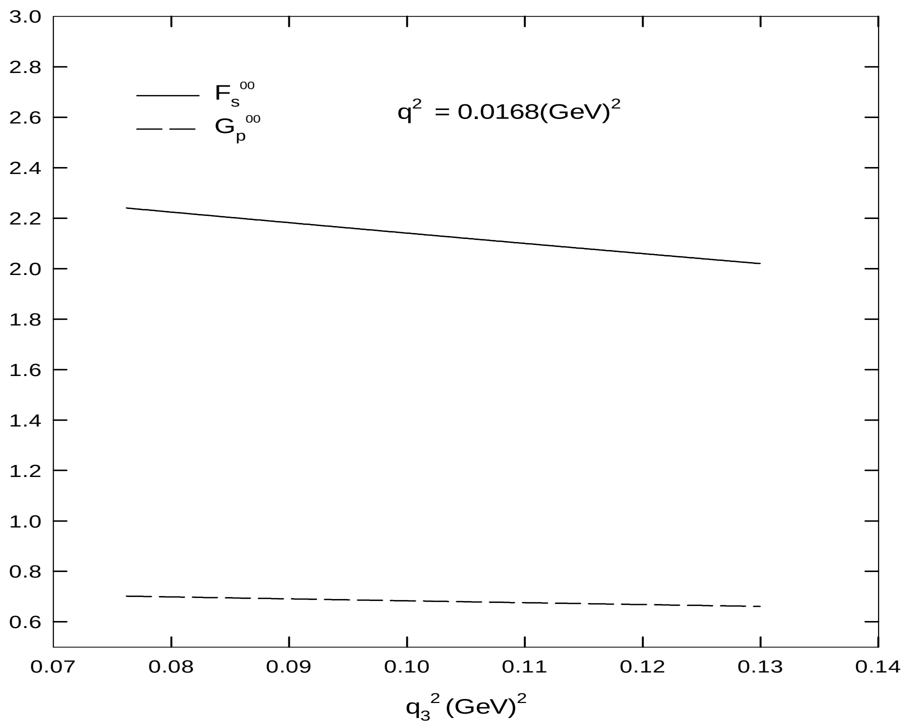
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
| F$_p^{+0}$ |
| G$_s^{+0}$ |
q_3^2 = 0.14(GeV)^2
0.00
0.01
0.02
0.03
0.04
0.05
q^2 (GeV)^2

Fig.9



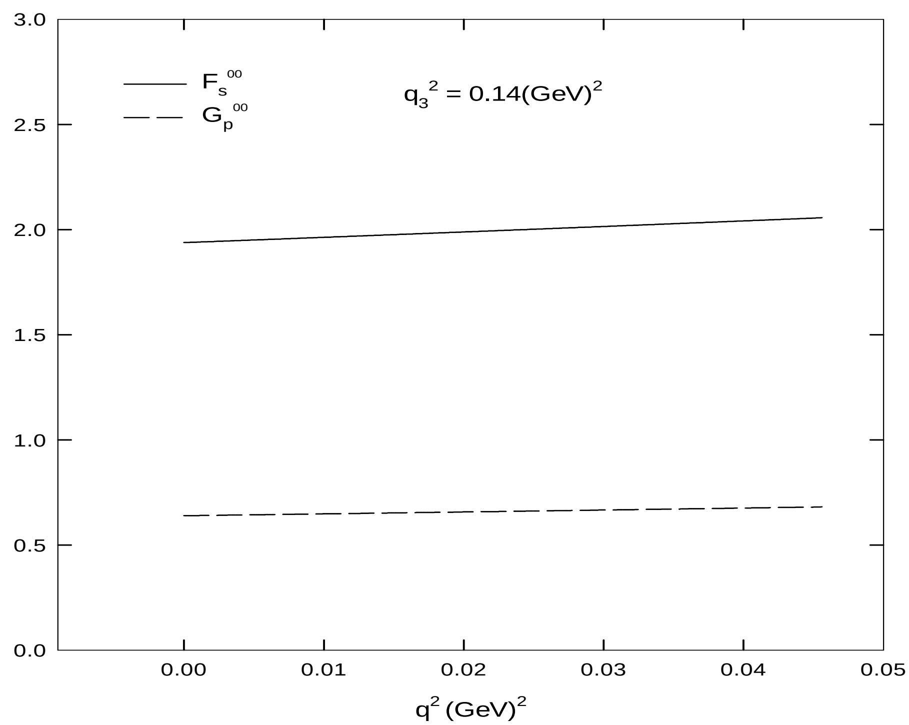
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
0.07
0.08
0.09
0.10
0.11
0.12
0.13
0.14
$|F_p^{+o}|$
$|G_s^{+o}|$
$q^2 = 0.0168(\mathrm{GeV})^2$
$q_3^2\ (\mathrm{GeV})^2$

Fig.10



3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
F_s^{oo}
G_p^{oo}
$q^2 = 0.0168(\mathrm{GeV})^2$
0.07
0.08
0.09
0.10
0.11
0.12
0.13
0.14
$q_3^2\,(\mathrm{GeV})^2$

Fig.11



F_s^{oo}
G_p^{oo}
$q_3^2 = 0.14(\mathrm{GeV})^2$
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.00
0.01
0.02
0.03
0.04
0.05
$q^2\,(\mathrm{GeV})^2$

Fig.12